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THE DIFFRACTION OF AN ARBITRARY ACOUSTIC WAVE BY A BLOCK†

P. V. TRET'YAKOV

Kaliningrad, Moscow Region

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The solution of the problem of the diffraction of an arbitrary acoustic wave by a block (including the spatial case) is considered. The same approach is used as in the case of diffraction by a wedge [1].

The problem of diffraction by a block has been considered in many publications [2–6]. However, the most detailed solution has only been written out for the first of the diffraction waves which arise at the corners of the block. When the subsequent waves (or the incidence of non-planar waves) is considered, the solution obtained using both Green's functions [2] as well as a radial expansion [3] is quite complicated. The results which are closest to those in this paper were obtained in [6] for the two-dimensional case.

Let us consider a block with the transverse cross-section shown in Fig. 1. Let the z_1-axis of the Cartesian system of coordinates $x_1y_1z_1$, associated with the first of the newly formed edges O_1 coincide with this edge. The x_1-axis is directed from the first edge to the second and the distance between the edges is equal to a. Finally, the y_1-axis is perpendicular to the first two and, together with them, forms a right-handed system of coordinates.

Let us introduce a system of coordinates $x_2y_2z_2$, associated with the second edge O_2, in the following manner: $x_2 = a - x_1$, $y_2 = y_1$, $z_2 = -z_1$.

Let an arbitrary acoustic wave be incident on the block. The excess pressure (or the velocity potential) behind this wave can be represented, in coordinates associated with the corresponding edge, in the form

$$f_j\left(t_{j1}, r_j, z_j, \cos(\alpha_j + \vartheta_j)\right) H\left(\eta_{j1}(\alpha_j + \vartheta_j)\right) \tag{1}$$

$$r_j^2 = x_j^2 + y_j^2, \quad \vartheta_j = \operatorname{arctg}\left(y_j / x_j\right)$$

Here t_{j1} is the time measured from the instant when the wave arrives at the edge O_j, $\alpha_j = \text{const}$ is the angle which the normal to the wave front at an instant $t_{j1} = 0$ makes with the plane passing through the edge of the block at the point O_j (Fig. 2) and $H(x)$ is the Heaviside function. The incident wave satisfies the wave equation

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \vartheta^2} + \frac{\partial^2 f}{\partial z^2} - \frac{\partial^2 f}{\partial t^2} = 0 \tag{2}$$

The solution of the diffraction problem must also satisfy the same equation.

The presence in (1) of a factor in the form of a Heaviside function means that the incident wave front can be represented in the form

$$\psi_{j1}\left(\alpha_j + \vartheta_j\right) = \eta_{j1}\left(t_{j1}, r_j, z_j\right) - \cos\left(\alpha_j + \vartheta_j\right) = 0$$

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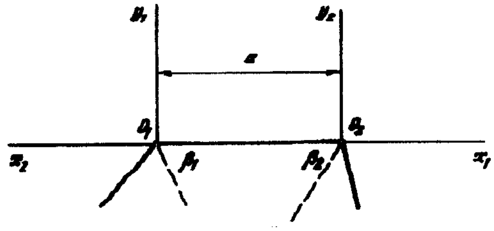

Fig. 1.

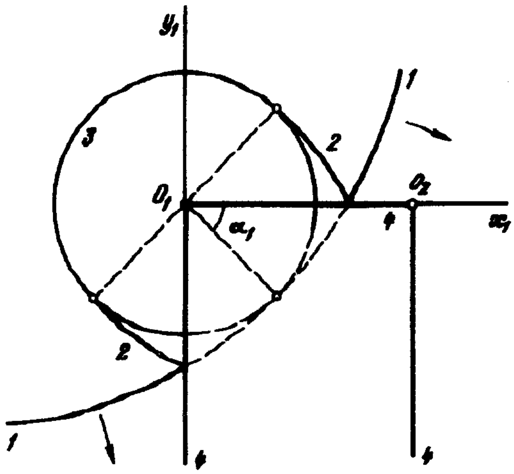

Fig. 2.

An impermeability condition $\partial f/\partial n = 0$ (an absolutely rigid screen) or $f = 0$ (an absolutely soft screen) is imposed on the surface of the block.

This problem was solved in [5, 6] in the two-dimensional case using a similar formulation and Leontovich boundary conditions on the surface of the wedge ($\partial f/\partial n = c\partial f/\partial t$). Furthermore, the solution of the problem of diffraction by an angle (wedge) was represented in the form of a double integral, the kernel of which $m(\beta, \varphi)$ was located under the sign of the inner integral. In this paper, a representation in the form of a single integral of the type of a Duhamel integral is used which simplifies the construction of the solution in the case of multiple diffraction. Moreover, the solution is generalized to the spatial case.

Diffraction by the edge O_j starts at the instant of time $t_{j1} = 0$. The diffraction surface is described by the relationship $\eta_{j1}(t_{j1}, r_j, z_j) = 1$.

Until the wave from one edge has reached the second edge, diffraction occurs as in the case of a wedge. A diagram of the diffraction in the $z_1 = z_2 = 0$ plane, which corresponds to this time interval, is shown in Fig. 2. In addition to the incident wave 1, the reflected wave 2 also participates in the diffraction and, moreover, irrespective of how many reflected waves exist in the physical plane (see [1]), there is only a single reflected wave in a mathematical sense. The diffracted wave is denoted in Fig. 2 by the number 3 and the block itself by the number 4. It has been shown [1] in the case of diffraction by a wedge that it is more convenient to treat the diffraction problem separately for the incident and reflected waves in a Riemann surface with a periodicity $4\pi - 4\beta_j$ with respect to ϑ_j, where β_j is the half angle of the wedge. The boundary conditions on the surface of the wedge are then satisfied using a linear combination of the resulting solutions. The same procedure will be used below.

Let a consider a Riemann manifold which corresponds to the block. For each of the edges (O_1 and O_2) there is a Riemann surface and, moreover, these surfaces are in communication with one another when $\vartheta_j = 2\pi k/\lambda_j$, where $k = 0, \pm 1, \pm 2, \dots, \lambda_j = 2\pi(4\pi - 4\beta_j)$, $j = 1, 2$ and a transition onto a new sheet of one of them at the above mentioned value of ϑ_j is also accompanied by a transition onto a new sheet by the other.

To simplify the discussion (without any loss of generality), let us consider the case when $\lambda_{1,2} = 2/3$ which corresponds to the block with right corners shown in Fig. 2. In this case, the surfaces are linked to one another when $\vartheta_j = 3\pi k$ $(j = 1, 2)$ and the periodicity of the solution in each of them will be 3π. An incident wave on Riemann surfaces exists when $-\alpha_j + 3\pi k \leqslant \vartheta_j \leqslant \alpha_j + 2\pi + 2\pi k$ $(k = 0, \pm 1, \pm 2, \ldots, j = 1, 2)$ which can be reflected on multiplying expression (1) for the incident wave by

$$H\left(\sin\left[\lambda_j\left(\alpha_j + \vartheta_j\right)/2\right]\sin\left[\lambda_j\left(2\pi - \alpha_j - \vartheta_j\right)/2\right]\right).$$

The solution of the problem of the diffraction of this wave at a Riemann surface with a periodicity $4\pi - 4\beta_j$ with respect to ϑ_j, which is added to the potential (1), has the form (see [1])

$$\varphi_{jk} = -\frac{1}{\pi}\int_{R_{jk}}^{1} f_j\left(t_{jk}, r_j, z_j, \upsilon\right)\left[g^+\left(u, \chi_j\right) + g^+\left(u, \mu_j\right)\right]du \tag{3}$$

$$\upsilon = \left(u^{1/\lambda} + u^{-1/\lambda}\right)/2, \quad R_{jk} = \left(\eta_{jk} - \sqrt{\eta_{jk}^2 - 1}\right)^{\lambda}, \quad \lambda = 2/3, \quad j,k = 1,2$$

$$\chi_j = \vartheta_j + \alpha_j, \quad \mu_j = 2\pi - \chi_j$$

$$g^+(u, \nu) = \sin\lambda\nu/\Delta(u, \nu), \quad \Delta(u, \nu) = 1 - 2u\cos\lambda\nu + u^2$$

At the instant of time $t_{11} + t_{21} = \alpha$, the diffracted waves, which arise at the different edges, intersect. The solution within the domain of intersection can be represented as

$$\Phi = f_j\left(t_{j1}, r_j, z_j\right)\cos\left(\alpha_j + \vartheta_j\right)H\left(\sin\left(\chi_j/2\right)\sin\left(\mu_j/2\right)\right)H\left(\psi_{j1}\left(\alpha_j + \vartheta_j\right)\right) + \Phi_{11} + \Phi_{21}, \tag{4}$$

$$\Phi_{jk} = \varphi_{jk}\left(t_{jk}, r_j, z_j, \vartheta_j\right)H\left(\eta_{jk} - 1\right)$$

and, in view of the fact that f_2 and f_1 are identical, we shall henceforth choose $j = 2$ or $j = 1$ depending on which representation is the more convenient for the treatment.

At the instant of time $t_{11} = \alpha$, the diffraction wave which arose at the first edge reaches the second edge and secondary diffraction starts to occur. The solution φ_{11} can be represented in the coordinates associated with the second edge in the form

$$\varphi_{11} = \bar{\varphi}_{11}\left(t_{22}, r_2, z_2, \vartheta_2 + \pi\right)H\left(\psi_{22}\left(\vartheta_2 + \pi\right)\right)$$

(t_{22} is the time measured from the instant when secondary diffraction occurs).

It has already been mentioned above that the Riemann surfaces communicate with one another when $\vartheta_j = 3\pi k$ and the wave φ_{11} will therefore not be present on each sheet as in the case of diffraction by a strip of finite width, but only when $-\pi + 3\pi k \leqslant \vartheta_2 \leqslant \pi + 3\pi k$. Consequently, all of the arguments put forward for the wave (1) will also be valid in the case of φ_{11}. However, $\vartheta_2 + \pi$ and not $\cos(\vartheta_2 + \pi)$ occurs as the argument in the latter case and the form of the solution in this case (see [1]) will be more complex

$$\varphi_{22} = \varphi_{22}^+ - i\varphi_{22}^-, \quad \varphi_{22}^{\pm} = -\frac{1}{2\pi}\int_{R_{22}}^{1} X_{22}^{\pm}\left(i\lambda^{-1}\ln u\right)\left[g^{\pm}\left(u, \vartheta_2 + \pi\right) + g^{\pm}\left(u, \pi - \vartheta_2\right)\right]du \tag{5}$$

Here

$$g^-(u, \nu) = (\cos\lambda\nu - u)/\Delta(u, \nu), \quad X_{22}^{\pm}\left(i\lambda^{-1}\ln u\right) = \bar{\varphi}_{11}\left(t_{22}, r_2, z_2, -i\lambda^{-1}\ln u\right) \pm \bar{\varphi}_{11}\left(t_{22}, r_2, z_2, i\lambda^{-1}\ln u\right) \tag{6}$$

and if $\varphi_{11}(t, r, z, \upsilon)$ is an analytic function with respect to the latter argument, then the functions on the right-hand side of (5) will be real, otherwise, it is necessary to retain just the real part.

The complete solution within the wave arising from secondary diffraction at the second edge is made up of the sum of the solutions for all of the waves which have passed through this domain, that is

$$\Phi = f_j\left(t_{j1}, r_j, z_j, \cos\left(\alpha_j + \vartheta_j\right)\right) H\left(\sin\left(\lambda\chi_j / 2\right)\sin\left(\lambda\mu_j / 2\right)\right) H\left(\psi_{j1}\left(\alpha_j + \vartheta_j\right)\right) + \Phi_{11}K_1 + \Phi_{21}K_2 + \Phi_{j2}$$

$$K_m = H\left(\sin\left[\lambda(\vartheta_m + \pi)/2\right]\sin\left[\lambda(\pi - \vartheta_m)/2\right]\right), \quad j = 2$$

The solution which is added to φ_{21} is constructed in precisely the same manner as in the case of secondary diffraction by the first edge. Only, in this case, the solution is constructed in the terms of coordinates associated with the first edge and $\overline{\varphi}_{12}(t_{12}, r_1, z_1, \upsilon)$ is used as the integrand in (5).

The process of the emergence of new diffraction waves can be considered up to infinity and each of the newly arising diffraction waves φ_{jk} can be represented in terms of the preceding $\overline{\varphi}_{lk-1}$ (here, $j = 1$ corresponds to $l = 2$, and $j = 2$ corresponds to $l = 1$) which appears at the opposite edge using a formula analogous to (5) when the indices 22 are replaced by jk. Here, instead of $\overline{\varphi}_{11}$ in formula (6), it is necessary to take the function φ_{lk-1} which is rewritten in terms of coordinates associated with the edge O_j and the time t_{jk} measured from the instant when the diffraction wave under consideration appears.

The complete solution in each of the diffraction domains is made up of the solutions for each of the waves which have passed through this domain.

Hence, the problem of the diffraction of an arbitrary wave of the form (1) in the Riemann manifold corresponding to the block is completely solved.

In order to obtain the solution of the same problem in the case of the reflected wave

$$f_j\left(t_{j1}, r_j, z_j, \cos\left(\alpha_j - \vartheta_j\right)\right) H\left(\psi_{j1}\left(\alpha_j - \vartheta_j\right)\right)$$

it is necessary to replace ϑ_j by $-\upsilon_j$ ($j = 1, 2$) in all of the constructions carried out above.

In order to determine the solution of the problem of diffraction by block, it is necessary to carry out the following procedure depending on the form of the boundary conditions.

If the impermeability condition $\partial f / \partial n = 0$ is satisfied, the solution for the reflected wave has to be added to the solution for the incident wave. If, however, the condition $f = 0$ is satisfied on the surface of the block, the solution for the reflected wave has to be subtracted from the solution for the incident wave.

The whole of the solution of the problem for a block with arbitrary corners (including the case when the corners are not equal) which has been expounded here can be repeated without causing any difficulties. This also applies to an acoustic wave of the form

$$f_j\left(t_{j1}, r_j, z_j, \alpha_j + \vartheta_j\right) H\left(\psi_{j1}\left(\alpha_j - \vartheta_j\right)\right)$$

The method which has been described can also be extended to the problem of the diffraction of an arbitrary acoustic wave by a cylinder with a transverse section in the form of a polygon.

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